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Oscillating Scalar Fields and Hubble Constants

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Abstract

We consider a recent proposal of Morikawa[1] that oscillations in the Hubble constant may be driven by coherent oscillations of an ultra–low mass scalar field (soft–boson). If the mass density of the Universe is dominated by soft–boson coherent oscillations then there will be an oscillating Hubble constant with period $\pi/m \equiv L = 20\,\mathrm{Mpc}/(m/10^{-30}\,\mathrm{eV})$ and amplitude $3H/4m = 0.04\,L/(100h^{-1}\,\mathrm{Mpc})$. We explore the particle physics implications and observational consequences of this proposal, which might have relevance for the regularity in the red shift distribution recently seen in a deep pencil beam survey.

Consider a massive, minimally coupled real scalar field ϕ with Lagrangian density

$$\mathcal{L}_\phi = (\partial_\mu \phi)^2/2 - m^2\phi^2/2. \tag{1}$$

If the scalar field is spatially homogeneous, then its stress-energy tensor takes on the perfect fluid form:[2]

$$T^\mu_\nu = \mathrm{diag}\,[\rho, -p, -p, -p]; \tag{2a}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2, \qquad p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2; \tag{2b}$$

where the metric tensor $g_{\mu\nu} = \mathrm{diag}\,[1, -R^2, -R^2, -R^2]$, $R(t)$ is the cosmic scale factor, and units are chosen so that $\hbar = c = 1$. The equation of motion for ϕ in a Robertson-Walker background space-time is

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0; \tag{3a}$$

$$\frac{d(R^3\rho)}{dt} = -p\frac{d(R^3)}{dt}; \tag{3b}$$

where $H \equiv \dot{R}/R$ is the expansion rate (Hubble parameter). Eq. (3a) which is the Klein-Gordon equation in an expanding space-time and Eq. (3b) which is the first law of thermodynamics are equivalent.

If we completely ignore the friction term due to the expansion of the Universe ($H \to 0$ or $m \gg H$), we see that ϕ oscillates with angular frequency m. Moreover, the average of the pressure p over one cycle is zero (just a statement of the virial theorem for a harmonic oscillator).[3] However, the average is achieved by the pressure oscillating between $-\rho$ and ρ. That is, if we write the equation of state as $p = \gamma\rho$, γ varies between -1 and 1!

Equation (3b) implies that the density of a perfect fluid with pressure $p = \gamma\rho$ evolves as $\rho \propto R^{-3(1+\gamma)}$, which gives the familiar results: $\rho \propto R^{-3}$ for $\gamma = 0$; $\rho \propto R^{-4}$ for $\gamma = 1/3$; $\rho \propto$ const for $\gamma = -1$; and the less familiar result that $\rho \propto R^{-6}$ for $\gamma = 1$. Thus, as the scalar field oscillates, its energy density decreases by a power of R that varies between 0 and -6, which on average is equal to -3. As we shall see shortly, such pressure oscillations can induce oscillations in the expansion rate of the Universe.

Let us assume that the mass density of the Universe is dominated by that of the scalar field oscillations—that is, the dark matter in the Universe exists in the form of coherent scalar field oscillations—then the expansion rate is given by

$$H^2 = \frac{8\pi G\rho}{3}; \tag{4}$$

where for simplicity we have assumed a flat Robertson-Walker model. The solution to the equation of motion for ϕ is simple to write down in the two limits $m \ll H$ and $m \gg 3H$.[2] In the limit $m \ll 3H$, where the oscillation time of the scalar field is longer than the Hubble time, the solution is $\phi = $ const—the scalar field ϕ remains "stuck" due to the friction that arises due to the expansion of the Universe. In the opposite limit, where the oscillation time is much less than the Hubble time, the solution is $\phi = \sqrt{2}A\cos mt/R^{3/2}$, where A determines the amplitude of the oscillations. In this limit the energy density $\rho = \dot{\phi}^2/2 + m^2\phi^2/2 \simeq A^2m^2/R^3$ decreases as R^{-3}: As mentioned earlier the coherent oscillations of a massive scalar field behave like nonrelativistic matter. More specifically, they correspond to a condensate of zero-momentum bosons of mass m.

Starting with this approximate solution, which is valid for $m \gg H$, and by assuming that the Universe is ϕ dominated so that to lowest order (in H/m) $R \propto t^{2/3}$, we can compute the $\mathcal{O}(H/m)$ corrections. To zeroth order $R^3\rho = $ const, and the oscillating pressure is given by

$$p = -\frac{A^2m^2\cos 2mt}{R^3} = -\,{}_0\rho\,\cos 2mt; \tag{5}$$

where $\rho = {}_0\rho + {}_1\rho$ and ${}_0\rho$ denotes the zeroth order solution: ${}_0\rho = A^2m^2/R^3$. By using Eq. (3b) we can solve for the first order correction to ${}_0\rho$:

$$\frac{{}_1\rho}{{}_0\rho} = \frac{\sin 2mt}{mt}. \tag{6}$$

Denote the zeroth-order Hubble constant as ${}_0H$, where ${}_0H^2 = 8\pi G\,{}_0\rho/3$. Using the above result for ${}_1\rho$ we can compute the first order correction to the Hubble constant:

$$H^2 = ({}_0H + {}_1H)^2 = \frac{8\pi G\,{}_0\rho}{3}\left(1 + \frac{{}_1\rho}{{}_0\rho}\right) \qquad \Rightarrow \quad {}_1H = \frac{3\,{}_0H}{4m}\,{}_0H \sin 2mt. \tag{7}$$

The Hubble constant oscillates with angular frequency $2m$ and amplitude $3_0H/4m$. Our linearized approach is strictly only valid for $H/m \ll 1$; however, the equations governing the evolution of ρ and H, Eqs. (3) and (4), are easily integrated for arbitrary ratio m/H. If coherent ϕ oscillations only contribute a fraction Ω_ϕ of the total energy density of the Universe, the amplitude of the Hubble constant oscillations is reduced by a factor of Ω_ϕ.

Are there any observable consequences of an oscillating Hubble constant? Yes! To begin, we remind the reader that these "Hubble oscillations" are spatially coherent throughout the Universe and are only oscillations in time. However, when we look out in space we look back in time, and so *any* observer in the Universe will observe a spherical wave pattern associated with the value of the Hubble constant. The distance to a nearby object (distance much less than H^{-1}) and the time t when the light we are observing today was emitted are related by: $t \simeq t_0 - d$, where t_0 denotes the present epoch. Thus, the Hubble constant inferred at distance d is

$$H(d) = H_0\left(1 + \frac{3H_0}{4m}\sin(\psi - 2md)\right), \tag{8}$$

where $H_0 = 100h\,\mathrm{km\,sec^{-1}\,Mpc^{-1}}$ denotes the present (zeroth order) value of the Hubble constant and $\psi = 2mt_0$ is the phase of the oscillations at the current epoch.

It is interesting to write $2md$ as $2md = 2\pi d/L$ where the oscillation length L is related to m by

$$L = \frac{\pi}{m} = \frac{20\,\mathrm{Mpc}}{m_{-30}},$$

and $m_{-30} = m/10^{-30}$ eV. The amplitude of the Hubble oscillations is

$$\frac{3H_0}{4m} \simeq \frac{1.6h \times 10^{-3}}{m_{-30}} \simeq 8 \times 10^{-3} \, \frac{L}{20h^{-1}\,\mathrm{Mpc}}.$$

In order for the Hubble oscillations induced by the oscillating scalar field to have a significant amplitude, the mass of the scalar must be very light indeed—a soft boson[4,5,6]; we will return later to address the plausibility of soft bosons in the context of modern particle theory.

Just what are the observable consequences? Since the value of the Hubble constant enters in essentially every cosmological test—e.g., galaxy number count-red shift test, luminosity-red shift test, angle-red shift test, etc.—Hubble oscillations should manifest themselves in many places![7] In order to be more specific, we need to derive the kinematic relations that hold in the presence of Hubble oscillations.

To begin, let's solve for the first order correction to the evolution of the cosmic scale factor; it is found by integrating

$$\frac{\dot{R}}{R} = H = \frac{2}{3}\frac{1}{t}\left[1 + \frac{\sin 2mt}{2mt}\right]. \tag{9}$$

Treating the oscillating term as a small perturbation, we find

$$R(t) = \left(\frac{t}{t_0}\right)^{2/3}\left[1 - \frac{3}{8}\frac{H_0^2}{m^2}\left(\cos 2mt - \cos\psi\right)\right], \tag{10}$$

where subscript zero denotes the present value of a quantity and $R_0 = 1$. Note that the correction to $R(t)$ is order H_0^2/m^2. The red shift z of a galaxy that emitted the light we see today at time t is given by

$$1 + z = R(t)^{-1} \simeq \left(\frac{t_0}{t}\right)^{2/3}\left[1 + \frac{3}{8}\frac{H_0^2}{m^2}\left(\cos 2mt - \cos\psi\right)\right]. \tag{10}$$

The luminosity-red shift (or Hubble) diagram is based upon the relationship between the luminosity distance of an object, $d_L^2 \equiv \mathcal{L}/4\pi\mathcal{F}$, and its red shift: $d_L =$

$r(1+z)$, where the observer's radial coordinate position is taken to be zero and that of the galaxy is r, $\mathcal{L}$ is the luminosity of the object, and $\mathcal{F}$ is the observed flux. The radial coordinate r of an object at red shift z is given by

$$r = \int_{t(z)}^{t_0} \frac{dt}{R(t)} \,,$$

where t is the time of emission. Integrating this equation and using Eq. (10), we find the key relationship between r and z

$$r = H_0^{-1}\left[z - \frac{3}{8}\frac{H_0^2}{m^2}\left(\cos[\psi - 2mH_0^{-1}z] - \cos\psi\right)\right] + \mathcal{O}(z^2) + \mathcal{O}(H_0^3/m^3), \qquad (11)$$

which is valid for small z. (We have dropped the higher order terms in z as we are most interested in the oscillating term in the "Hubble law.") Using Eq. (11) to express r in terms of z, we arrive at the modified Hubble law, valid for small z:

$$z = H_0\, d_L\left[1 + \frac{3}{8}\frac{H_0}{m^2 d_L}\left(\cos[\psi - 2md_L)] - \cos\psi\right)\right] + \mathcal{O}(z^2). \qquad (12)$$

The correction to the Hubble law (i.e., $z = H_0 d_L$) is periodic in space, with period $L = \pi/m$. The amplitude is more subtle to estimate. Suppose for simplicity that $\cos\psi = 0$. An object at a distance of $L/2 = \pi/2m$ has a red shift given by

$$z = \frac{H_0\pi}{2m}\left(1 - \frac{3}{4\pi}\frac{H_0}{m}\right);$$

that is, the correction due to the oscillating Hubble constant is order H_0/m.

The angle distance to an object d_A is defined as $d_A = D/\theta$, where D is its physical diameter of the object and θ is the angle that the object subtends on the sky. The angle-red shift relationship is: $d_A = r/(1+z) = d_L/(1+z)^2$. To lowest order in z the angle-red shift relationship is the same as the luminosity-red shift relationship, and so Eq. (12) with the substitution $d_L \to d_A$ provides the angle-red shift relationship.

Now, consider the number of galaxies per solid angle per red shift interval, $dN_{gal}/dzd\Omega$ (number count-red shift test). Assuming that the number density of galaxies per comoving volume element ($= n_c$) is constant, the number galaxies dN_{gal} in a solid angle $d\Omega$ is given by

$$dN_{gal} = n_c r^2 dr d\Omega;$$

to obtain $dN_{gal}/dz d\Omega$ from this expression we must relate r to z and dr to dz—which we can do by using Eq. (11). Doing so we find

$$\frac{dN_{gal}}{z^2 dz d\Omega} = H_0^{-3} n_c \left[1 - \frac{3}{4}\frac{H_0}{m} \sin[\psi - 2mH_0^{-1}z] - \frac{3}{4}\frac{1}{z}\frac{H_0^2}{m^2}\left(\cos[\psi - 2mH_0^{-1}z] - \cos\psi\right)\right], \tag{13}$$

where as before we have assumed $z \ll 1$. For small z, the usual result is $dN_{gal}/z^2 dz d\Omega = H_0^{-3} n_c$; the lowest order correction is periodic (period L) and has an amplitude of order $3H_0/2m$.

To this point we have assumed that the observer is at rest with respect to the cosmic rest frame (defined by the cosmic microwave background radiation); our galaxy moves with respect to the cosmic rest frame with a speed of about $2 \times 10^{-3} c$. If the observer is moving with respect to the cosmic rest frame with velocity $\mathbf{v}_P$, then the red shifts measured by this observer will differ from those measured by an observer at rest with respect to the cosmic rest frame by

$$z = z_v + \mathbf{v}_P \cdot \hat{\mathbf{n}} = z_v + v_P \cos\theta \qquad (z \ll 1),$$

where $\hat{\mathbf{n}}$ is the unit vector in the direction of the galaxy whose red shift in the moving frame is z_v. If we are only interested in the periodic effects, then the transformation to the moving frame is simple: in al previous expressions z is replaced by $z_v + v_P \cos\theta$, valid for $z \ll 1$. The net effect is that the argument of all the sinusoidal terms becomes:

$$\psi - 2mH_0^{-1} z \rightarrow \psi(\theta) - 2mH_0^{-1} z, \tag{14}$$

where the direction dependent phase $\psi(\theta)$ is given by

$$\psi(\theta) = \psi - \frac{2m}{H_0} v_P \cos\theta \simeq \psi - 2\pi \left(\frac{6h^{-1}\,\mathrm{Mpc}}{L} \frac{v_P}{2 \times 10^{-3} c} \cos\theta \right). \tag{15}$$

Due to the peculiar velocity of the observer, the apparent center of the spherical pattern is shifted; however, the period is not changed.

The crucial assumption that underlies Hubble oscillations is that of the coherence of the scalar field oscillations. If they do not remain spatially coherent, then, averaged over all of space, the equation of state $p = 0$ will be a very good approximation, and Hubble oscillations will not arise. While one might worry that spatial coherence would be both difficult to arrange initially and difficult to maintain in the presence of density inhomogeneities, these worries may not be so serious. First, the initial spatial coherence may be "arranged" by inflation: Provided that the scalar field is smooth in some small region at the beginning of inflation, inflation will enlarge that region to encompass the present Hubble volume. Second, owing to the uncertainty principle it is difficult for the field to become inhomogeneous on scales less than about $300m^{-1} \sim 100L$. The uncertainty principle implies that $\Delta p \Delta x = m \Delta v \Delta x \gtrsim 1$; the deepest potential wells in the observable Universe are those of rich clusters and are characterized by velocity dispersions of less than or of order of $v_{max} \sim 3 \times 10^{-3}c$. Thus the uncertainty principle precludes the collapse of soft bosons on scales smaller than about $(v_{max}m)^{-1} \sim 300m^{-1}$, or about $100L$. This fact would also help to reconcile the theoretical prejudice for $\Omega = 1$, with the experimental reality that the fraction of critical density contributed by material clustered on scales less than about 30 Mpc only contributes $\Omega_{30} \simeq 0.2$, as soft bosons would necessarily be smooth on these scales and would not contribute to dynamical determinations of the mass density of the Universe.

Perhaps a more important concern is whether the effects of an oscillating Hubble constant are large enough to be observed. Since $m \gtrsim 3H$ is required for the scalar field to begin oscillating,[2] the maximum amplitude of the Hubble oscillations is only about 24%. Of course, our analysis here is linear and nonlinear effects could increase the amplitude of the oscillations and their effects. In any case, Hubble oscillations are certainly novel enough to bear discussion and just as importantly have a "smoking gun" kind of signature: A spherical wave pattern around each observer in the Universe, and a simple relationship between the Hubble oscillation length and amplitude,

amplitude of $8 \times 10^{-3}\,(L/20h^{-1}\,\mathrm{Mpc})$. A most interesting question is whether or not such oscillations could have anything to do with the periodic distribution of red shifts seen by Broadhurst et al.[8] in their deep (to red shift $z \sim 0.5$), pencil-beam survey of the North and South Galactic Pole regions. The spatial period they infer is about $120h^{-1}$ Mpc; for such a period, we would predict that the amplitude of the oscillatory term in $dN_{gal}/z^2dzd\Omega$ to be about 10%.

Finally, let us discuss the plausibility of such a scenario: Can a well motivated particle physics model lead to such a light scalar field? The idea of ultra-low mass scalar fields has recently been a subject of very active discussion. The familiar "invisible axion" is a natural realization of this idea,[9] but with a mass $m_a \sim \Lambda_{QCD}^2/f_a \sim 10^{-5}$ eV that is much larger than is of interest here ($f_a \sim 10^{12}$ GeV is the scale of Peccei-Quinn symmetry breaking). Note that by "naturalness" we mean the "Harvard" definition of the late 1970's: A Lagrangian $\mathcal{L}(\mu, M)$ with parameters μ and M satisfying $\mu \ll M$ is natural provided that $\mathcal{L}(0, M)$ corresponds to a symmetry limit in which $\mu = 0$ is protected by the symmetry to all orders of perturbation theory. For example $m_{up} \ll m_{top}$ is natural since a chiral symmetry protects $m_{up} = 0$. Setting $\Lambda_{QCD} \to 0$ corresponds to $\partial_\mu S^\mu = T^\mu_\mu \propto \beta(g) \to 0$, where S^μ is the dilatation current; hence scale invariance is recovered and protects $\Lambda_{QCD} = 0$. Hence, naturalness is essentially a symmetry principle, and may or may not have anything to do with aesthetics. A technically natural generalization of axions and familons has been proposed[4] in which $m_\phi \sim m_{fermion}^2/f$. If the fermion mass corresponds to that of a light neutrino, $m_{fermion} \sim m_\nu \sim 0.01$ eV, and $f \sim 10^{16}$ GeV, then one obtains the astrophysically interesting mass scale of order 10^{-29} eV. This idea has been exploited as the basis of a model to generate large-scale structure[5] and to account for the dark matter as coherent field oscillations.[6]

The oscillating Hubble constant scenario was proposed originally by Morikawa.[1] Morikawa emphasizes the general non–minimally coupled case, in which the Lagrangian for Φ contains a term of the form $\xi\Phi^2\mathcal{R}$. He argues that the amplitude of the resulting Hubble oscillations can be made consistent with the observations reported by Broadhurst et al.[8] However, on the basis of the criterion of naturalness, we would argue that such a model is unlikely. The only way to make the mass naturally as small as $\sim 10^{-30}$ eV is to exploit pseudo-Goldstone bosons of some kind, yet such fields will not admit $\xi\Phi^2\mathcal{R}$ terms. We might expect induced terms of the form $\sim (m^4/M_{Pl}^2)\cos(\Phi/f)\mathcal{R}$ but these would have negligible effects. It should be emphasized that if one demands a $\xi\Phi^2\mathcal{R}$ term with $\xi\Phi/MPl$ having large amplitude excursions, then one has an effective time–dependent variation in G_{Newton}. Oscillations of the gravitational constant sufficient to explain the Broadhurst et al.[8] results may be inconsistent with the very stringent solar system limits to the time variation of G_{Newton}.[10] We will return to this question elsewhere.

One might have hoped to make Morikawa's model natural by identifying the field Φ with $M_{Pl}e^{\phi/M_{Pl}}$ where ϕ is the dilaton field, a field that expresses a nonlinearly realized conformal invariance at the highest energies. The dilaton is expected to arise in superstring theories and couples to the curvature scalar like a Brans–Dicke field. However, the mass scale expected for the dilaton is, at the very least, given by the trace anomaly of QCD (and probably involves higher energy scales). Thus the dilaton mass is expected to be bounded below by a scale of order $\Lambda_{QCD}^2/m_{Pl} \sim 10^{-11}$ eV, (and is probably much heavier) and therefore is unlikely to correspond to a cosmologically interesting scale.[11]

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References

1. M. Morikawa, Univ. of British Columbia preprint 90-0208.

2. Coherent scalar field oscillations in the context of the expanding Universe are discussed by M.S. Turner, *Phys. Rev. D* **28**, 1243 (1983) and by E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), pp. 275-277.

3. The average of the pressure over one cycle for a potential of the form $V(\phi) = \beta\phi^n$ is $p= (n-2)\rho/(n+2)$; for a more complete discussion of coherent scalar field oscillations in the expanding Universe see Ref. 7.

4. C.T. Hill, G.G. Ross, *Nucl. Phys.* **B311**, 253 (1988); *Phys. Lett. B* **203**, 125 (1988).

5. C.T. Hill, D.N. Schramm, and J.N. Fry, *Comments Nucl. and Part. Phys.* **19**, 25 (1989); M.S. Turner, A. Stebbins, *Astrophys. J.* **339**, L13, (1989); W.Press, B.S. Ryden, and D. Spergel, *Phys. Rev. Lett.* **64**, 1084 (1990).

6. W. Press, B.S. Ryden, and D. Spergel, *Phys. Rev. Lett.* **64**, 1084 (1990).

7. For a discussion of the standard kinematic cosmological tests, see Kolb and Turner in Ref. 1 (Chapter 2), S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chapter 15, or A. Sandage, *Astrophys. J.* **133**, 355 (1961).

8. T.J. Broadhurst, R.S. Ellis, D.C. Koo, and A.S. Szalay, *Nature* **343**, 726 (1990).

9. J. Preskill, M. Wise, and F. Wilczek, *Phys. Lett. B* **120**, 127 (1983); L. Abbott and P. Sikivie, *ibid*, 133 (1983); M. Dine and W. Fischler, *ibid*, 137 (1983).

10. R.D. Reasenberg et al., *Astrophys. J.* **234**, L219 (1979).

11. R.D. Peccei, J. Sola, and C. Wetterich, *Phys. Lett. B* **195**, 183 (1987).